STRENGTH in NUMBERS
tutoring and test prep

## STRENGTH IN NUMBERS BRIDGE CLASS

## CALCULUS (COVERAGE OF MOST AP TOPICS, AB AND BC)

### TEXTBOOK - *CALCULUS: EARLY TRANSCENDENTALS (6TH EDITION) BY JAMES STEWART.*

**This class is a comprehensive introduction to the calculus of a function of a single real variable.** Over the course of this class you will learn various techniques for computing derivatives, integrals, and their applications.

**What is "Calculus?"** Over the course of your math studies, you have learned about many different topics. For the past couple of years you have probably learned to divide math into "algebra" and "geometry." Calculus is another form of math, more like algebra than geometry.

**Calculus concerns itself with the concept of a "function,"** just like algebra. However, algebra and calculus treat functions differently. Algebra generally has to do with the *individual values* of a function, so we might ask "what is f(2)?" or "what are the zeros of this function over the real numbers?" Calculus, however, asks different questions about functions, generally having to do with how the functions *change*. So in calculus, we ask questions like "what is the *rate of change* of this function when x=2?" or "how many times does the graph of this function reach a local maximum over the real numbers?" For reasons that will be made clear as the course unfolds, we can use these same techniques – albeit in reverse – to also answer questions about certain "totals" achieved by functions over time intervals. (In calculus, rates of change are usually called "derivatives" and computing them is called "differentiation." Totals are usually called "integrals," and computing them is called "integrating." Much of the time and effort involved in learning calculus has to do with applying techniques of differentiation and integration to increasingly complex and exotic functions.)

**Essentially all of calculus involves extension of concepts from algebra.** Algebra can, for example, tell us the slope of a line connecting *two distinct* points on a curve, but it cannot provide us with the slope of a curve *at a single point* along the curve. Using some clever techniques from algebra, however, it is possible to turn the new problem of "slope at a point" into the familiar, old problem of "slope over an interval." Take heed: almost all of the math you learn from this point on will involve using techniques you have already mastered to tackle increasingly more complex problems. A lot of math consists of taking new, unfamiliar problems and turning them into old, familiar problems that you already understand and know how to solve.

**The topics we will cover in the class will roughly correspond to the following topic list.** I have placed the topics in order according to which class will be used to cover them. Brief descriptions of the topics are included, as well as corresponding textbook sections. **Homework will be assigned at the time of the class; all problems, whenever possible, will be problems for which answers exist in the back of the book and therefore checking the answers will be easy.** There will be some more "project-based" type problems which have no computational answer and whose answer instead depends on writing out a mathematical argument. These are the really good problems!

**<u>Note: This class is deliberately planned in an ambitious way.</u>** You will need to work hard and show up ready to put in significant effort for each class. There is no reason you cannot succeed as long as you put in the requisite amount of effort; the class is structured so that someone with an affinity and gift for mathematics can master its content in the time allotted. Calculus is not easy, but it is absolutely possible to learn quickly and intuitively. Be patient with yourself and expect to make lots of mistakes at first. A new way of thinking will always be tough while you're getting used to it, but as time goes on, your mistakes will decrease in severity and number. The great 20th century mathematician Paul Halmos famously said that "the only way to learn mathematics is to *do* mathematics." But to do, at first, is to do imperfectly—and it is better to begin imperfectly than not begin at all.

| Class number | Topics | Book Sections |
|---|---|---|
| 1 | Derivative as a function; limit definition of the derivative. | 2.7, 2.8, ch. 2 review |
| 2 | Derivatives cont'd | 2.7, 2.8, ch. 2 review |
| 3 | Derivatives of polynomials, exponential functions, trigonometric functions; product and quotient rules. | 3.1, 3.2, 3.3 |
| 4 | Chain rule; implicit differentiation; derivatives of logarithms. | 3.4, 3.5, 3.6 |
| 5 | Related rates; max+min problems; mean value theorem. | 3.9, 4.1, 4.2 |
| 6 | Derivatives and graphing; l'Hôpital's rule; curve sketching. | 4.3, 4.4, 4.5 |
| 7 | Optimization; review of content thus far; brief intro to antiderivatives. | 4.7, 4.9 |
| 8 | Intro to integration; area as "reverse rate of change" for functions. | 5.1, 5.2, 5.3 |
| 9 | Integration by substitution; areas between curves; volumes. | 5.5, 6.1, 6.2 |
| 10 | Volumes using shells; average value of a function. | 6.3, 6.5 |
| 11 | Integration by parts; partial fractions. | 7.1, 7.4 |
| 12 | Trigonometric integrals; trigonometric substitution. | 7.2, 7.3 |
| 13 | Improper integrals; integrating "to infinity" or over vertical asymptotes; review of content thus far. | 7.8, ch. 7 review |
| 14 | Arc length; discussion of integrals as tools for finding totals. | 8.1 - extension problems with line/surface integrals, if possible |
| 15 | Introduction to infinite series, integral test for convergence. | 11.2, 11.3 |
| 16 | Comparison test, alternating series; ratio and root tests for convergence. | 11.4, 11.5, 11.6 |