STRENGTH in NUMBERS
tutoring and test prep

STRENGTH IN NUMBERS BRIDGE CLASS

GEOMETRY: FOUNDATIONS OF PROOF

TEXTBOOK - *CME PROJECT GEOMETRY, COMMON CORE (PEARSON).*

Forget all of the prior work you've done in algebra that said it was "geometry" when really it was just making you solve equations involving shapes. Those problems can be interesting, but we will be doing very little of that.

But if geometry isn't solving equations, what *is* it? It's really about learning the basics of what it means for something to be "true" in math. For instance, say I give you the following problem:

- *Here is a picture of a circle. Its center is not labeled, and neither is its radius; you are given only its curved edge. Can you locate the center of the circle?*

If you have only studied algebra, at this point you will not have a rigorous method for finding the center. You might take measurements and try to find a unique point that appears to be the same distance from the edge in any direction. But this, unfortunately, isn't good enough. We need a way to *prove*, in a way that other people are *forbidden to disagree with,* that the point we locate is the exact center of the circle. No quibbling about whether we made our measurements precisely enough. We aren't finding a "picture" of the center, but rather a precise, infallible method for locating its center *in principle*. This is what geometry is: the ability to become a mathematical "truth chef," producing logical arguments that prohibit objections.

Geometry is a highly dialectical subject, meaning that it is best discovered by discussion and exploration instead of by explicit instruction alone. The main goal in geometry is to comprehend the logical *foundations* for what is being done, not merely to memorize a list of formulas. Since geometry is so highly dependent on argumentation instead of calculation, this class will almost seem like an English or history class at times. You will need to know lots of vocabulary and be capable of integrating a large body of existing knowledge. Each time that we learn something new, that new thing becomes a tool we can use to solve future problems. For example, say that we are able to crack the problem about circles above, and come up with a reliable, foolproof way to find the center. In future problems, if we ever have occasion to locate the center of an otherwise-unmarked circle, we will have a method ready to do so! Sometimes when we go to solve a problem, we will need to use techniques that we haven't used for a long time. There is no way around this—the topics in geometry don't proceed in a straight line (**pause for laughter**) and sometimes the tools we learn earlier on will be used later in unexpected ways. For this reason, completing problems is valuable in geometry the same way it was valuable in algebra: it helps you to internalize current concepts, so as to apply them more easily in the future.

Anyway, that's the end of the boring intro that always goes at the start of these things, even though nobody ever reads it! Here is the part you care about: the schedule.

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In each section, we will work through a certain number of problems during class. These problems will be intended to apply the concepts we define and discuss during that day's class. The remainder of each problem set will be left as homework.

Class number	Topics	Book Sections
1	Compasses, angles, and circles; geometry basics and notation	p. 30 1,4,7-13,16
2	Geometry software; drawings and constructions	p. 35 1-8 p. 37 1-8 p. 41 1-8
3	UnMessUpable figures (drawn with geometry software); invariants; intro to congruence	p. 80 1-8, 11
4	Triangle congruence; SAS, ASA, etc.	p. 87 1-8, plus try to prove PBT.
5	Proof; converse of PBT; angle bisector equidistant theorem; parallel lines	p. 101 1-8 p. 108 1-10
6	Deduction and proof; parallel lines	p. 113 6-7 p. 116 6, 8 p. 122 9, 11, 12 p. 124 4-9 p. 128 1-9, 11
7	Continued work on triangles; introduction to quadrilaterals	p. 130 7-8 p. 140 18-19 p. 146 5,9 p. 149 11 p. 152 8 p. 156 31-32
8	Areas of basic quadrilaterals; midline theorem for trapezoids; Pythagorean theorem	p. 178 5 p. 191 5-8 p. 221 1-11 p. 227 6 (the diagonal described is the **longest** one in the cube, the one that goes from one corner, through the middle of the cube, all the way to the opposite corner. This is sometimes called the "space diagonal," since it travels through three-dimensional space instead of just on a plane.)
9	Intro to solids; surface area; volume; prisms, cylinders and cones; spheres	p. 232 1-6 (how do 5 and 6 relate to solids?) p. 238 5-8 p. 243 5-10, p. 244 1,4,6,7,10-11 p. 251 1-8

Class number	Topics	Book Sections
10	Scale factors and similarity; side-splitter theorems	p. 270 Discussion Q's 6-7, Check Your Understanding 7-8, 16-18 p. 319 4-6
11	Similar figures; Tests for similar triangles; connecting area and circumference	p. 371 5-11 (it is ok if we need to discuss some of these in the next class; some of them are hard.)
12	Theorems about circles	p. 404 5-7, 10 p. 411 4, 6, 9, 11-13
13	Secants and tangents	Section: "Secants and Tangents," Check Your Understanding #5-7, 9-12. (The weird "y" looking symbol in problem 12 is the lowercase Greek letter gamma, in case you're wondering. It's not super important to use that exact symbol, obviously.)
14	Power of a point	p. 433 1-7 p. 450 5 p. 453 6, 8
15	Trigonometry; Law of Sines	6.8 Check Your Understanding 1-4, 6, 11, 19-20
16	Extension of the Pythagorean theorem into the Law of Cosines	6.9 Check Your Understanding 1-5, 8 Also see final challenge problem below, in bold!

Final challenge problem! We saw using the Law of Sines that sometimes, triangles can have more than one solution. When this was true, it was because the triangles had side-side-angle style data, which can generate more than one triangle since SSA isn't a congruence postulate. Draw some diagrams and explore why Law of Cosines *cannot* give more than one solution. The easy answer is "because it doesn't use SSA data," and that's a fine answer. But think also about how we *found* the alternate solution for Law of Sines. (We did it by reflecting the angle over the y-axis to find the other angle that had the same sine as the one we found.) If you try to find an "alternate solution" for Law of Cosines, it will involve finding another angle that has the same cosine. How can you find an angle that has the same *cosine* as a given angle? (Hint: to find one with the same *sine*, we flipped over the y-axis, so…) How does this help you conclude that Law of Cosines does not permit "alternate" solutions?

A final note on difficulty, and the value of working through something on your own.

Sometimes (perhaps quite often), you'll find a problem that will be very difficult, and you might need to peek at a solution or look something up. That's not bad, but you should make sure that before doing so, you're really stumped. Getting frustrated and not knowing what to do is a normal and valuable part of the learning process. When you do look something up to help you, try to use the source only until you can resume on your own. If a proof is supposed to have 10 steps and you're stuck on step six, you could look at a solution to see what is done at *that* step, but then try subsequent steps on your own. There are no math police to check that you are doing this, but it serves your own interests if you handle as much work as possible on your own.