STRENGTH *in* NUMBERS
tutoring and test prep

## STRENGTH IN NUMBERS BRIDGE CLASS

## MULTIVARIABLE CALCULUS: THE CALCULUS OF VECTORS

TEXTBOOK - *CALCULUS: EARLY TRANSCENDENTALS (6TH EDITION) BY JAMES STEWART.*

**In single-variable calculus** (AB, BC or "Calc 1 and 2" if you're in college), you learned how to differentiate and integrate functions of a single real variable. In multivariable calculus, you will extend these concepts to functions of more than one variable. As a very basic example, whereas in single-variable calculus we calculated the *area* bounded by a one-dimensional *curve* and the *x-axis*, in multivariable calculus we will learn how to calculate the *volume* bounded by a two-dimensional *surface* and the *xy-plane*. The ability to do this has many more interesting payoffs than just being able to do that, though.

**The class will begin with some stuff that doesn't seem like calculus:** we will be talking about "vectors," which you might recognize (or might not) from a physics or other science class. The reason we begin talking about vectors is that when we think of a function *f(x,y)* being evaluated at "two" values, one each for *x* and *y*, it will often be simpler to regard the pair (*x,y*) as a single object and call it a "vector" than to think of each number as its own separate entity. Accordingly, we will spend some time developing the idea of a vector and some basic things we can do with them. The point of this is getting familiar enough with vectors that we can treat them as inputs of functions just as comfortably as we have customarily treated real numbers.

**Some of the results in this class will seem obvious; that's because some of them are.** Some of them will seem difficult; that's because some of them are. Problems and concepts will be made significantly easier if you develop the ability to draw decent pictures of what you're dealing with. This can be hard, because a lot of what we are going to learn about is three-dimensional, and there is a reason most mathematicians didn't go to art school. But just the same way that drawing pictures made it easier to define a derivative, or to define a Riemann sum, there will be certain concepts in this class (including the extensions of derivatives and Riemann sums to higher-dimensional analogues!) that will get a heck of a lot more intuitive if you know how to draw a picture.

**Here is the list of topics we will be covering and the problem sets to go with them.** Some of the later problem sets will not have very many problems in them. That's because problems involving later concepts tend to take a long time to solve, and in the interest of being reasonable with time commitments, I'm assigning a small number. Engaging with the later concepts will be fruitful even with small numbers of problems, so don't worry. Earlier on in the class, I will be assigning larger numbers of problems, but this is because they will be more routine and easier to complete; it also is because the concepts developed early on will be foundational for later understanding, so it will be especially important to get them firmly in your head through repetition.

**You might need to look up help for some problems from time to time.** That's fine, but if you find yourself relying heavily on outside sources, you should mention it to me because it means we need to review fundamentals more. Don't worry about derailing our topical coverage if we need to do this. It is better to learn 4 chapters thoroughly than to learn 6 chapters badly. Also, if you're looking things up, try to look up only enough information to clear whatever intellectual logjam is preventing you from solving your current problem, and then trying to solve the rest on your own. More than that, and you risk creating an atmosphere of dependence on "sources" instead of your own reasoning. The goal here is understanding, and completion of problem sets is only fruitful when put at the service of that higher goal.

| Class number | Topics | Homework |
|---|---|---|
| 1 | Vectors; dot products; orthogonality | p. 777 #17,19,21-23 (all, not just odd),24,35<br>p. 784 # 1-9 odd, 15-19 odd, 23-24<br>p. 792 #1-7 odd, 13-21 odd |
| 2 | Cross products; equations of lines and planes | p. 802 #7-11 odd, 23-31 odd, 49 |
| 3 | Vector functions and space curves; parametrization; derivatives and integrals of vector functions | p. 822 #19-23 odd, 25<br>p. 828 #9-19 odd, 33-37 odd |
| 4 | Functions of several variables; level curves & level surfaces; limits and continuity | p. 865 #7-19 odd<br>p. 877 #7,13,19,31,33,**39,41**(**these two** deal with polar coordinates, emphasized here because they will become important later) |
| 5 | Partial derivatives; equality of mixed partials; directional derivatives | p. 888 #15-27 odd, 57, 59<br>p. 920 #7-15 odd, 23, 25 |
| 6 | Max and min values; general catch-up | p. 930 #7-19 odd, 31, 33 |
| 7 | Double integrals over rectangles; iterated integrals; Fubini's theorem | p. 964 #3-17 odd, 29, 31 |
| 8 | Double integrals over general regions; double integrals in polar coordinates | p. 972 #7-17 odd, 45-49 odd<br>p. 978 #7-19 odd, 25-31 odd |
| 9 | Triple integrals; cylindrical coordinates; spherical coordinates | p. 998 #13-23 odd<br>p. 1004 #1,3,5,9, 17-21 odd, 27<br>p. 1010 #1,3,5,9,17-25 odd (sketch optional for #17, but may be helpful) |
| 10 | Change of variables; general catch-up | p. 1020 #1-11 odd, 23 (challenge problem) |
| 11 | Vector fields; conservative vector fields; line integrals | p. 1032 #11-18 (all, not just odd), 21-25 odd<br>p. 1043 #1-7 odd, 17 |
| 12 | Line integrals of vector fields; Fundamental theorem for line integrals | p. 1043 19, 21<br>p. 1053 #3-15 odd, 19 |

| Class number | Topics | Book Sections |
|---|---|---|
| 13 | Green's theorem; curl and divergence; identifying nonconservative vector fields | p. 1060 #1-7 odd, 13<br>p. 1068 #1-7 odd, 13-19 odd |
| 14 | Parametric surfaces and their areas; surface integrals | p. 1091 #5-9 odd, 19, 21 |
| 15 | Stokes' theorem | p. 1097 #1,3,7,9,13 |
| 16 | Divergence theorem | p. 1103 #1,3,7,11 |

Please note that most problem sets consist entirely of odd-numbered problems. This is because the answers to these questions are in the back of the book, so you can check your work. I have sometimes included some even-numbered questions as well, when they were either elementary enough that I don't think you'll need to check the answers, or too interesting to pass up.

This class will be a lot of work, but you can do it. Usually the pattern in Calculus (and this class is no exception) is that you learn a new topic, rehearse the basics, and then dive deep on the details. When this happens, I've tried to assign a relatively high number of "routine" problems initially, to get you familiar with new concepts like line integrals or cross products, but then switch to a lower volume of more difficult problems later on.

Enjoy! This is going to be fun.